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Question Paper Code : 80218

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulation 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let A and B be two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and $P(A\cap B)=\frac{1}{4}$. Compute $P(A/B)$ and $P(\overline{A}\cap\overline{B})$.

2. A random variable X has probability mass function $P(X=x)=\frac{x}{10}$, $x=1, 2, 3, 4$. Find the cumulative distribution function, $F(x)$ of X.

3. The joint probability density function of (X, Y) is $f(x, y)=\begin{cases}4xy, & 0\le x\le 1,\ 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$. Calculate $P(X\le 2Y)$.

4. State the Central limit theorem for independent and identically distributed random variables.

5. The random process $X(t)$ is defined as $X(t)=e^{-B|t|}$ where B is a random variable uniformly distributed over $[0, 2]$. Is $X(t)$ a first order stationary?

6. Let $\{X_n : n\ge 0\}$ be a Markov chain having state space $S=\{1, 2\}$ and one-step transition probability matrix given by $P=\begin{bmatrix}1 & 0\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$. Find the stationary probabilities of the Markov chain.

7. In an $(M/M/1/\infty):FCFS$ queueing system, the arrival rate is $\lambda=3$ customers/minute and utilization ratio $\rho=0.5$. Find L_s and W_s.

8. In an $(M/M/C/N)$ $(C<N)$ FCFS queueing system, write expressions for P_0 and P_N.

9. In an $M/D/1:FCFS$ queueing system, an arrival rate of customers is 10 per second and a service rate of customers is 20 per second. Compute the mean number of customers in the system.

10. Consider a two-station tandem Markovian queueing network with customers arrival rate of $\lambda = 2$/minute and service rates $\mu_1 = 4$/minute at station-1 and $\mu_2 = 6$/minute at station-2. Compute the waiting time of a customer in the system and the probability that both the servers are idle.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A bag contains 3 black and 4 white balls. Two balls are drawn at random one at a time without replacement. (1) What is the probability that the second ball drawn is white? (2) What is the conditional probability that the first ball drawn is white if the second ball is known to be white?

(ii) Suppose the random variable X has a geometric distribution $P(X=x)=\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{x-1}$, $x=1, 2, 3, \ldots$. Determine (1) $P(X \le 2)$ (2) $P(X>4/X>2)$ (3) Moment generating function, $M_X(t)$ of X and hence obtain $E(X)$ and $Var(X)$.

Or

(b) (i) A consulting firm rents cars from three rental agencies in the following manner: 20% from agency D, 20% from agency E and 60% from agency F. If 10% cars from D, 12% of the cars from E and 4% of the cars from F have bad tyres, what is the probability that the firm will get a car with bad tyres? Find the probability that a car with bad tyres is rented from agency F.

(ii) If a random variable X has the probability density function,

$$f(x)=\begin{cases} x e^{-x}, & x>0 \\ 0, & x \le 0, \end{cases}$$

find (1) Moment generating function, $M_X(t)$ of X and hence obtain $E(X)$ and $Var(X)$ (2) Cumulative distribution function of X (3) $P(X \le 2)$.

12. (a) (i) Let the joint probability mass function of the random variables (X, Y) be

$$P(X=x, Y=y)=\begin{cases} \frac{x+y}{12}, & x=1,2, \; y=1, 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find (1) the marginal probability mass functions of X and Y (2) $P(X+Y \le 3)$ (3) $P(X>Y)$ (4) Are the R.Vs X and Y independent?

(ii) The joint probability density function of (X, Y) is

$$f(x, y) = \begin{cases} Cxy^2, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine (1) the value of 'C' (2) the p.d.f. conditional of X given that $Y = y$ (3) $E(XY)$.

Or

(b) (i) Let X and Y be discrete R.Vs with joint probability mass function

$$P(X = x, Y = y) = \begin{cases} \dfrac{x+y}{21}, & x = 1, 2, 3, \ y = 1, 2 \\ 0, & \text{otherwise.} \end{cases}$$

Compute the correlation coefficient, ρ_{XY}, of X and Y.

(ii) Two random variables X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the p.d.f. of the R.V. $U = \dfrac{X}{Y}$.

13. (a) (i) A random process is given as $X(t) = U + V\cos(\omega t + \theta)$ where U is a R.V. with $E(U) = 0$ and $Var(U) = 3$, V is a R.V. with $E(V) = 0$ and $Var(V) = 4$, 'ω' is a constant and 'θ' is a R.V. with p.d.f. $f(\theta) = \dfrac{1}{2\pi}$, $-\dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2}$. It is further assumed that U, V and θ are independent R.Vs. Is the process $X(t)$ stationary in the wide-sense? Explain.

(ii) Let $\{X_n : n \ge 0\}$ be a Markov chain having state space $S = \{1, 2, 3\}$ and one-step TPM $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$. (1) Draw a transition diagram for this chain. (2) Is the chain irreducible? Explain. (3) Is the state-3 ergodic? Explain.

Or

(b) (i) Let $X(t)$ and $Y(t)$ be two independent Poisson processes with parameters λ_1 and λ_2, respectively. Compute (1) $P(X(t)+Y(t)=n), n=0,1,2,\ldots$ (2) $P(X(t)-Y(t)=n), n=0,\pm1,\pm2,\ldots$

(ii) A random process has sample function of the form $X(t)=A\cos(\omega_0 t+\theta)$ where ω_0 is a constant. 'A' is a R.V. that has a magnitude of +1 and –1 with equal probability and 'θ' is a R.V. that is uniformly distributed over $[0, 2\pi]$. Assume that the random variables 'A' and 'θ' are independent. Is $X(t)$ a wide-sense stationary process? Explain.

14. (a) (i) Patients arrive at a clinic according to a Poisson process at a rate of 30 patient per hour. The waiting room cannot accommodate more than 14 patients. Examination time per patient is exponentially distributed random variable with rate of 20 per hour. (1) Find the effective arrival rate at the clinic. (2) What is the probability that an arriving patient will not wait? (3) What is the expected waiting time until a patient is discharged from the clinic?

(ii) For an $M/M/1/\infty:FCFS$ queueing system, derive the system of differential-difference equations for the system size probabilities. Obtain the corresponding steady-state equations and hence calculate the steady-state probabilities of the system size and its the mean system size.

Or

(b) (i) A petrol pump station has 4 pumps. The service time follows an exponential distribution with mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Compute (1) the probability that an arriving car will have to wait in the system (2) the average waiting time W_q in the queue (3) the mean number, L_s, of cars in the system.

(ii) For an $M/M/1$ balking queue, derive the steady-state probabilities of the system size by assuming that the service rate as $\mu_n=\mu$, $n=1,2,3,\ldots$ and the arrival rate of the customers as $\lambda_n=\frac{\lambda}{n+1}$, $n=0,1,2,\ldots$ where 'n' is the number of customers in the system.

15. (a) Discuss an $M/G/1/\infty:FCFS$ queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce also the mean number of customers for the $M/M/1/\infty:FCFS$ queueing system from the P-K mean value formula.

Or

(b) Discuss the Jackson open queueing network system and hence obtain the corresponding product form solution of the system size probabilities.

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